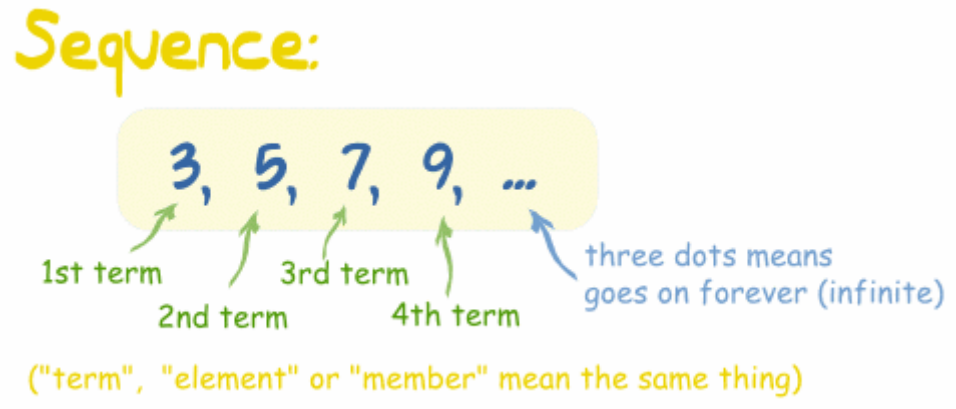
Sequences

What is a sequence?	Is a set of numbers that are in order (pattern) We can use algebra to describe those patterns.



Arithmetic sequences	An Arithmetic Sequence is made by **adding** some value each time.

1, 4, 7, 10, 13, 16, 19, 22, 25, ...
This sequence has a difference of 3 between each number.
The pattern is continued by adding 3 to the last number each time.
3, 8, 13, 18, 23, 28, 33, 38, ...
This sequence has a difference of 5 between each number.
The pattern is continued by adding 5 to the last number each time.

The value added each time is called the "**common difference**"

The common difference could also be negative, like this:

25, 23, 21, 19, 17, 15, ...
This common difference is **-2**
The pattern is continued by **subtracting 2** each time.

Geometric Sequences	A Geometric Sequence is made by **multiplying** by some value each time.

2, 4, 8, 16, 32, 64, 128, 256, ...
This sequence has a factor of 2 between each number.
The pattern is continued by multiplying the last number by 2 each time.

3, 9, 27, 81, 243, 729, 2187, ...

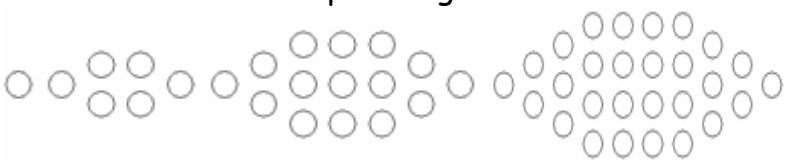
This sequence has a factor of 3 between each number.
The pattern is continued by multiplying the last number by 3 each time.
The value that you multiply each time is called "**common ratio**"
The common ratio could be also less than 1 , 0,25, for example, in this case you have to divide by 4 in order to continue the sequence.

Triangulars Numbers	1, 3, 6, 10, 15,...
Square Numbers	1, 4, 9, 16,...
Cube Numbers	1, 8, 27, 64, ...
Fibonacci Numbers	1, 1, 2, 3, 5, 8, 13, 21, ...

All of them are special sequences
Can you figure out the **next** few numbers?

Make your own Number Patterns

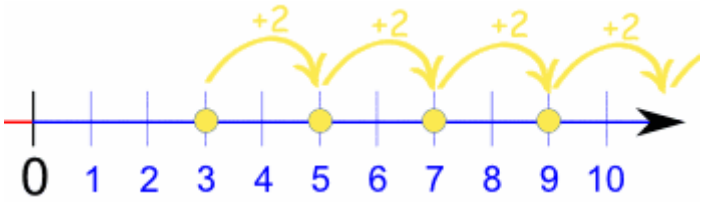
You can make your own number patterns using coins or matchsticks. Here is an example using coins:



Size=1 Size=2 Size=3 Size=4
1 Coin 6 Coins 15 Coins 28 Coins

How many coins would you need when Size=5 ?
Can you make a formula that will tell you how many coins are needed for any Size?
For example Size=20? The formula may look something like
Coins = Size × Size + ...

Finding a rule



Saying " starts at 3 and jump 2 every time" is a good description, but you have to calculate the :

- 10 th term
- 100 th term, or
- nth term, where **n** could be any term number we want.

So, we want a formula with n to calculate any term of the sequence.

So, What Would A Rule For 3, 5, 7, 9, ... Be?

Firstly, we can see the sequence goes up 2 every time, so we can **guess** that a Rule will be something like "2 times n" (where "n" is the term number). Let's test it out:

Test Rule: 2n

n	Term	Test Rule
1	3	2**n** = 2×**1** = **2**
2	5	2**n** = 2×**2** = **4**
3	7	2**n** = 2×**3** = **6**

That **nearly** worked ... but it is **too low** by 1 every time, so let us try changing it to:

Test Rule: 2n+1

n	Term	Test Rule
1	3	2**n**+1 = 2×**1** + 1 = 3
2	5	2**n**+1 = 2×**2** + 1 = 5
3	7	2**n**+1 = 2×**3** + 1 = 7

That Works!

So instead of saying "starts at 3 and jumps 2 every time" we write this:

$a_n = 2n + 1$

Now we can calculate, for example, the **100th term**:

$a_{100} = 2 \times 100 + 1 =$ **201**

Example: Calculate the first 4 terms of this sequence:

$$a_n = (-1/n)^n$$

Calculations:

- $a_1 = (-1/1)^1 = -1$
- $a_2 = (-1/2)^2 = 1/4$
- $a_3 = (-1/3)^3 = -1/27$
- $a_4 = (-1/4)^4 = 1/256$

Answer:

-1, 1/4, -1/27, 1/256, ...

Arithmetic Sequences Rule

In an arithmetic sequence the difference between one term and the next is a constant.

In other words, you just add some value each time ... on to infinity.

Example:

1, 4, 7, 10, 13, 16, 19, 22, 25, ...

This sequence has a difference of 3 between each number.
Its Rule is an = 3n-2

In General you could write an arithmetic sequence like this:

{a, a+d, a+2d, a+3d, ... }

where:

- a is the first term, and
- d is the difference between the terms (called the "common difference")

And you can make the rule by:

- an = a + d(n-1)

(We use "n-1" because **d** is not used in the 1st term).

Geometric Sequences Rule

In a geometric sequence each term is found by **multiplying** the previous term by a **constant**.

Example:

2, 4, 8, 16, 32, 64, 128, 256, ...

This sequence has a factor of 2 between each number.

Its Rule is $x_n = 2^n$

In General you could write an arithmetic sequence like this:

{a, ar, ar^2, ar^3, ... }

where:

- **a** is the first term, and
- **r** is the factor between the terms (called the **"common ratio"**)

  Note: **r** should not be 0 or 1.

When **r=0**, you get the sequence {a,0,0,...} which is not geometric

When **r=1**, you get the sequence {a,a,a,...} which is not geometric

And the rule is:

- $a_n = a\, r^{(n-1)}$

  (We use "n-1" because a r^0 is the 1st term)

Exercises

- What is the twenty-first term of the sequence given by $x_n = 4n - 3$?
- What is the fifth term of the sequence $a_n = (1/n)^{n-1}$?
- Calculate the tenth term of the sequence $a_n = -2n + 10$
- Work out the formula for this sequence -2, 1, 4, 7, ...
- Work out the formula for this geometric sequence 100, 50, 25, 12.5, 6.25,....
- Copy the following arithmetic progressions into your notebook and work out the **general term:**

3, 7, 11, 15, ... -12, -9, -6, -3, ... 12, 9, 6, 3, ... 10, 3, -4, 11, ... 120, 152, 184, ...

- Copy the following geometric progressions into your notebook and work out the **general term:**

1, 3, 9, 27, 81, ...; -5, -10, -20, -40, ... ;**1024, 512, 256, ...** ; **100, 150, 225, ...**